1.4 Building Functions from Functions

Let $f(x)=x^2$ and $g(x)=\sqrt{x+1}$

D: $(-\infty, \infty)$ $[-1, \infty)$

$\sqrt{x+1}^2 \geq 0^2$

$x+1 \geq 0$

$-1 \quad -1$

$x \geq -1$

Find the following and give the domain of each new function.

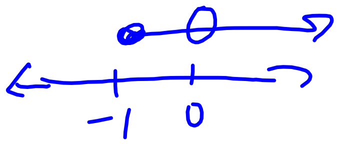
f+g

$x^2+\sqrt{x+1}$ $[-1, \infty)$

f−g

$x^2-\sqrt{x+1}$ $[-1, \infty)$

f/g $\dfrac{x^2}{\sqrt{x+1}}$ $(-1, \infty)$



g/f $\dfrac{\sqrt{x+1}}{x^2}$ $[-1, 0) \cup (0, \infty)$

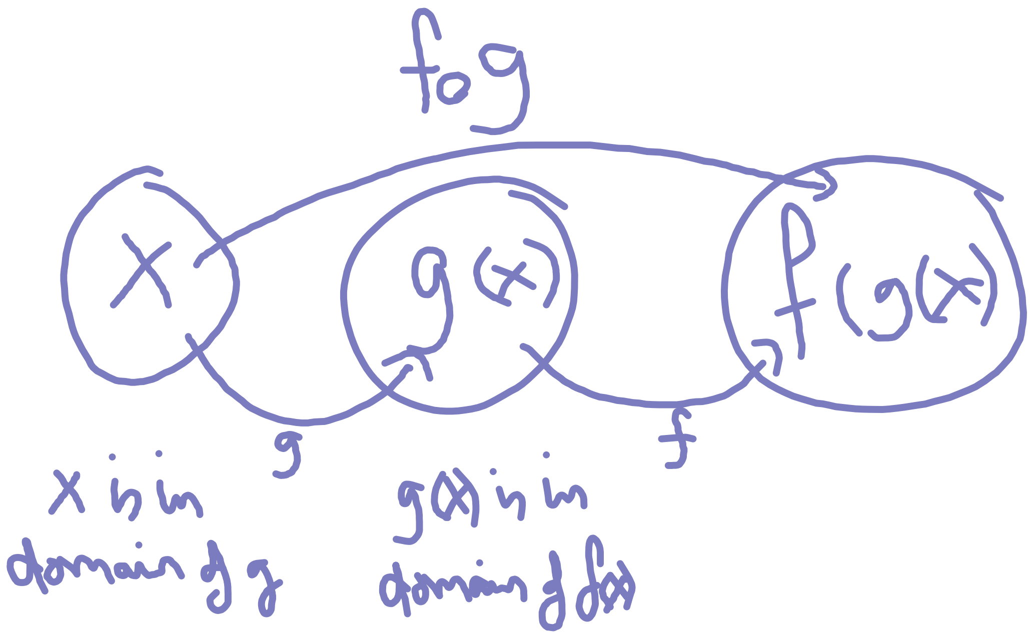
gg $\sqrt{x+1}\sqrt{x+1}$ $[-1, \infty)$ inherited domain

$= x+1$

When combining functions, the composition $f \circ g$ is denoted:

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ consists of all x-values in the domain of g that map to $g(x)$ values in the domain of f.



$f(g(x)) = f \circ g(x)$

Ex.1 Let $f(x)=2^x$ and $g(x)=\sqrt{x}$. Find $f \circ g(x)$ and $g \circ f(x)$ and. Verify the functions $f \circ g$ and $g \circ f$ are not the same.

$f(x) = 2^x$. D: $(-\infty, \infty)$ $g(x) = \sqrt{x}$ D: $[0, \infty)$

$f(g(x)) = 2^{\sqrt{x}}$ $[0, \infty)$

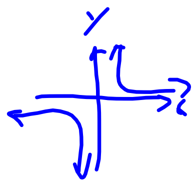
parent child

$g(f(x)) = \sqrt{2^x}$ $(-\infty, \infty)$

The domain of $f \circ g$ is defined for $[0, \infty)$.
The domain of $g \circ f$ is defined for $(-\infty, \infty)$.

We could also verify graphically.

Find $f(g(x))$ and $g(f(x))$ for



$f(x) = 2x^2$ $g(x) = \frac{1}{x}$

D: $(-\infty, \infty)$ $(-\infty, 0) \cup (0, \infty)$

$f(g(x)) = 2\left(\frac{1}{x}\right)^2$

$= \frac{2}{x^2}$ $(-\infty, 0) \cup (0, \infty)$

$g(f(x)) = \frac{1}{2x^2}$ $(-\infty, 0) \cup (0, \infty)$

Ex.2

Find $f\circ g(2)$ and $g\circ f(-3)$ given $f(x)=x^2+2x-1$ and $g(x)=x-4$

$f(g(2))$

$g(2) = 2-4$

$= -2$

$f(-2) = (-2)^2+2(-2)-1$

$= 4-4-1$

$= -1$

$f(g(2)) = -1$

$g(f(-3))$

$f(-3) = (-3)^2+2(-3)-1$

$= 9-6-1$

$= 2$

$g(2) = 2-4$

$= -2$

TRY

Find $g\circ f(4)$ and $f\circ g(-2)$

given $f(x)=\sqrt{\cancel{x}}-3$ and $g(x)=3x^2-12$

Ex.3

Find the domain of the composite functions $g\circ f$ and $f\circ g$

given $f(x)=x^2-1$ and $g(x)=\sqrt{\cancel{x}}$

D: $(-\infty,\infty)$ D: $[0,\infty)$

$g(f(x)) = \sqrt{x^2-1}$

$(-\infty,-1] \cup [1,\infty)$

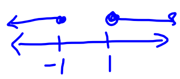
fits into domain of child.

$\left(\sqrt{x^2-1}\right)^2 \geq 0^2$

$x^2-1 \geq 0$

$x^2 \geq 1$

$x \geq 1$ or $x \leq -1$



$f(g(x)) = (\sqrt{x})^2 - 1$

$= x-1$ D: $[0,\infty)$

TRY

Find the domain of the composite functions $g\circ f$ and $f\circ g$

given $f(x)= 1/(x-1)$ and $g(x)=\sqrt{x}$.

Decomposing Functions

Ex.4 If h(x)= $\frac{1}{x^2-9}+4(x^2-9)^3$ find functions f(x) and g(x) such that h(x)=f(g(x)).

TRY If $h(x)=-2(x^2-1)^3+x^2-1$ find functions f(x) and g(x) such that h(x)=f(g(x)).

Intro to Calc

Using Implicitly Defined Functions

Ex.5

Describe the graph of the relation $9x^2 - 12xy + 4y^2 = 16$.

Ex.6

Describe the graph of the relation $x^2 + 7y^2=84$.

Ex.7

Describe the graph of the relation $x+3|y|=9$